Unit: 2. CIRCLES

POSTULATES AND THEOREMS

Postulates, corollaries, and theorems are in the order they appear in the curriculum.

Postulate 1: A line contains at least two points.

Postulate 1a: A plane contains at least three points not all on one line.

Postulate 1b: Space contains at least four points not all in one plane.

Postulate 2: Through any two different points, exactly one line exists.

Postulate 3: Through any three points that are not on one line, exactly one plane exists.

Postulate 4: If two points lie in a plane, the line containing them lies in that plane.

Postulate 5: If two planes intersect, then their intersection is a line.

Postulates of Addition:

Closure: the sum $a + b$ is a real number

Commutative: $a + b = b + a$

Associative: $a + (b + c) = (a + b) + c$

Addition of zero: $a + 0 = 0 + a = a$

Additive inverse: $a + -a = -a + a = 0$

Postulates of Multiplication:

Closure: the product, ab is a real number

Commutative: $ab = ba$

Associative: $(ab)c = a(bc)$

Multiplication by one: $a \times 1 = 1 \times a = a$

Multiplication inverse: $a \times 1/a = 1/a \times a = 1$

Distributive: $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$

Theorem 1-1: If two lines intersect, then their intersection is exactly one point.

Theorem 1-2: Exactly one plane contains a given line and a given point not on the line.

Theorem 1-3: If two lines intersect, then exactly one plane contains both lines.

Properties of equalities:

Addition: If $a=b$, then $a + c = b + c$ and $c + a = c + b$

Subtraction: If $a = b$, then $a - c = b - c$ and $c - a = c - b$

Multiplication: If $a = b$, then $ac = bc$

Division: If $a = b$ and c not $= 0$, then $a/c = b/c$

Properties on inequalities:

Subtraction: If $a<b$, then $a-c < b-c$

Division: If $a < b$ and $c > 0$, then $a/c < b/c$

If $a < b$ and $c < 0$, then $a/c > b/c$

Substitution: If a = b, a may be replaced by b, and b

by a in

any equation or inequality

Zero Product: If ab = 0, then a = 0 or b = 0, or a and b = 0

Postulate 6: Every angle corresponds with a unique real number greater than 0 degrees and less than 180 degrees (Angle Measurement Postulate).

Postulate 7: The set of rays on the same sign of a line with a common endpoint in the line can be put in one to one correspondence with the real numbers from 0 degrees to 180 degrees inclusive in such a way.

Theorem 3-2: If the exterior sides of two adjacent angles are opposite rays, then the angles are supplementary.

Theorem 3-3: If two lines are perpendicular, then they form right angles.

Theorem 3-4: If two adjacent angles have their exterior sides in perpendicular sides, then the angles are complementary.

Theorem 3-5: If two angles are supplementary to the same angles or two equal angles, then they are equal to each other.

Theorem 3-6: If two angles are complementary to the same angle or two equal angles, then they are equal to each other.

Theorem 3-7: If two lines intersect, the vertical angles formed are equal.

Theorem 3-8: All right angles are equal.

Theorem 3-9: If two lines meet and form right angles, then the lines are perpendicular.

Theorem 3-10: If two parallel planes are cut by a third plane, then the lines of intersection are parallel.

Postulate 8: If two parallel lines are cut by a transversal, then the corresponding angles have equal measure.

Theorem 3-11: If a transversal is perpendicular to one of the two parallel lines, then it is perpendicular to the other line also.

Theorem 3-12: If two parallel lines are cut by a transversal, then the alternate interior angles are equal.

Theorem 3-13: If two parallel lines are cut by a transversal, then the alternate exterior angles are equal.

Postulate 9: Through a point not on the line, one and only one line can

be drawn parallel to the line.

Postulate 10: If two lines are cut by a transversal so that corresponding angles are equal, then the lines are parallel.

Theorem 3-14: In a plane, if two lines are perpendicular to a third line, then they are parallel to each other.

Theorem 3-15: If two lines are cut by a transversal so alternate interior angles are equal, then the lines are parallel.

Theorem 3-16: If two lines are cut by a transversal so that alternate exterior angles are equal, then the lines are parallel.

Theorem 3-17: The sum of the measures of the angles of a triangle is 180 degrees.

Corollary 1: If two angles of one triangle are equal to two angles of another triangle, then the third angles are also equal.

Corollary 2: Each angle of an equiangular triangle is equal to 60 degrees.

Corollary 3: A triangle has at most one right angle or one obtuse angle.

Corollary 4: Acute angles of a right triangle are complementary.

Theorem 3-18: The measure of an exterior angle of a triangle is equal to

the sum of the measure of the remote interior angle.

Theorem 3-19: The sum of the measure of the angles of a quadrilateral is 360 degrees.

Postulate 11: If three sides of one triangle are equal to three sides of another triangle, then the triangles are congruent (SSS postulate).

Postulate 12: If two sides and the included angle of one triangle are

equal to two sides and the included angle of another triangle, then the triangles are congruent (SAS Postulate).

Postulate 13: If two angles and the included side of one triangle are equal to two angles and the included side of another triangle, then the triangles are congruent (ASA Postulate).

Theorem 4-1: If two angles and a not included side of one triangle are equal to the corresponding parts of another triangle, then the triangles are congruent (AAS Theorem).

Theorem 4-2: If two legs of one right triangle are equal to two legs of another right triangle, then the two right triangles are congruent (LL Theorem).

Postulate 14: If the hypotenuse and a leg of one right triangle are equal to the hypotenuse and leg of another right triangle, then the triangles are congruent (HL Postulate).

Theorem 4-3: If the hypotenuse and an acute angle of one right triangle are equal to the hypotenuse and an acute angle of another right triangle, then the triangles are congruent (HA Theorem).

Theorem 4-4: If a leg and an acute angle of one right triangle are equal

to the corresponding parts of another right triangle, then the triangles are congruent (LA Theorem).

Theorem 4-5: The altitude to the base of an isosceles triangle bisects the base.

Theorem 4-6: The base angles of an isosceles triangle are equal.

Theorem 4-7: The altitude to the base of an isosceles triangle bisects the vertex angle of the triangle.

Theorem 4-8: If two angles of a triangle are equal, then the sides opposite those angles are equal.

Theorem 4-9: If two sides of a triangle are not equal, then the angle opposite the longer side is the larger angle.

Theorem 4-10: If two angles of a triangle are not equal, then the side opposite the larger angle is the longer side.

Theorem 4-11: The sum of the length of any two sides of a triangle is greater than the length of the third side. (Triangle Inequality Theorem)

Theorem 4-12: If two sides of one triangle are equal to two sides of another triangle but the included angle of the first is larger than the included angle of the second, then the third side of the first triangle is longer than the third side of the second angle.

Theorem 4-13: If two sides of one triangle are equal to two sides of another triangle, but the third side of the first angle is longer than the third side of the second triangle, then the included angle of the first angle is larger than the included angle of the second triangle.

Theorem 4-14: If a diagonal is drawn in a parallelogram, then two congruent triangles are formed.

Theorem 4-15: The diagonals of a parallelogram bisect each other.

Theorem 4-16: If two sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.

Theorem 4-17: If both pairs of opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.

Theorem 4-18: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Theorem 4-19: If you connect the midpoints of two sides of a triangle, the resulting segment is parallel to the third side and equals one-half the length of the third side.

Theorem 4-20: The diagonals of a rectangle are equal.

Theorem 4-21: The diagonals of a rhombus are perpendicular.

Theorem 4-22: Each diagonal of a rhombus bisects two angles of a rhombus.

Theorem 4-23: The median of a trapezoid is parallel to the bases, and its length is one-half the sum of the lengths of bases.

Theorem 4-24: The base angles of an isosceles trapezoid are equal.

Theorem 4-25: The diagonals of an isosceles trapezoid are equal.

Postulate 15: If three angles of one triangle are equal to three angles of another triangle, then the triangles are similar (AAA for Similar Triangles).

Corollary 1: If a line intersects two sides of a triangle and is parallel to the third side, then a triangle similar to the given triangle is formed.

Corollary 2: Triangles similar to the same triangle are similar to each other.

Theorem 5-2: If two sides of one triangle are proportional to two sides of another triangle and the included angles are equal, then the triangles are similar (SAS for Similar Triangles).

Theorem 5-3: If three sides of one triangle are proportional to three sides of another triangle, then the triangles are similar (SSS for Similar Triangles).

Theorem 5-4: Similarity of polygons is reflexive, symmetric, and transitive (Transitive Part Proof).

Theorem 5-5: If two polygons are similar, then the ratio of their perimeters equals the ratio of any pair of corresponding sides.

Theorem 5-6: If a line is parallel to one side of a triangle and intersects the other two sides, it divides them proportionally.

Theorem 5-7: If a ray bisects an angle of a triangle, it divides the opposite side into segments with lengths proportional to the lengths of the other two sides of the triangle.

Theorem 5-8: If two triangles are similar, then the length of corresponding altitudes have the same ration as the length of any pair of corresponding sides.

Theorem 5-9: If the altitude to the hypotenuse of a right triangle is drawn, then the two triangles formed are similar to each other and similar to the given triangle.

Corollary 1: The length of a leg of a right triangle is the geometric mean between the length of the hypotenuse and the length of the projection of that leg on the hypotenuse.

Corollary 2: The length of the altitude to the hypotenuse is the geometric mean between the length of the segments of the hypotenuse.

Corollary 3: If a right triangle, the product of the hypotenuse and the altitude to the hypotenuse is equal to the product of the lengths of the legs.

Theorem 5-10: In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of hypotenuse (Pythagorean Theorem) $a^2 + b^2 = c^2$

Theorem 5-11: If the sum of the square of two sides of a triangle equals the square of the third side, then the triangle is a right triangle.

Theorem 5-12: In a 30, 60, 90 degree triangle, the hypotenuse is two times the short leg, and the longer leg is the short leg times the square root of three.

Theorem 5-13: If each acute angle of a right triangle has a measure of 45 degrees, then the measure of the hypotenuse is the square root of two times as long as a leg (45, 45, 90 Theorem).

Theorem 6-1: A radius drawn to a point of tangency is perpendicular to the tangent.

Theorem 6-2: A line in the plane of a circle and perpendicular to a radius at its outer endpoints is tangent to the circle.

Postulate 16: If the intersection of AB and BC of a circle is the single point B, then the measure of AB + the measure of BC = the measure of ABC (Arc Addition Postulate).

Theorem 6-3: If in the same circle or congruent circles the two central angles are equal, then their arcs are equal.

Theorem 6-4: If in the same circle or congruent circles the two minor arcs are equal, then their central angles are equal.

Theorem 6-5: In the same circle or congruent circles the chords are equal, then the arcs are equal.

Theorem 6-6: In the same circle or congruent circles if the arcs are equal, then their chords are equal.

Theorem 6-7: If a diameter is perpendicular to a chord, then it bisects the chord and its two arcs.

Theorem 6-8: In the same circle or in congruent circles if the chords are equidistant from the center, then their lengths are equal.

Theorem 6-9: In the same circle or in congruent circles if the chords have the same length, then they are equidistant from the center.

Theorem 6-10: The measure of an inscribed angle is equal to one-half the measure of its intercepted arc.

Corollary 1: An angle inscribed in a semicircle is a right angle.

Corollary 2: The opposite angles of an inscribed quadrilateral are supplementary.

Corollary 3: If two angles intercept the same or equal arcs, then the angles are equal.

Theorem 6-11: The measure of an angle formed by a secant ray and a tangent ray drawn from a point of a circle is equal to one-half the measure of its intercepted arc.

Theorem 6-12: The measure of an angle formed by two secants that intersect inside the circle is equal to one-half the sum of the intercepted arcs.

Theorem 6-13: The measure of the angle formed by two secants intersecting outside the circle equals one-half the difference of the intercepted arcs.

Theorem 6-14: The measure of the angle formed by a tangent ray and a secant ray intersecting outside the circle is equal to one-half the difference of the intercepted arcs.

Theorem 6-15: The measure of the angle formed by two tangent rays intersecting outside the circle is one-half the difference of the intercepted arcs.

Theorem 6-16: If two chords intersect in a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the other chord.

Theorem 6-17: If two secants intersect at a point outside a circle, then the length of one secant times the length of its external part is equal to the length of the other secant times the length of its external part.

Theorem 6-18: If a tangent segment and a secant segment intersect outside a circle, then the length of the tangent segment is the geometric mean between the secant segment and the external part of the secant segment.

Postulate 17: For every polygonal region there exists one unique positive number that is called the area of that region.

Postulate 18: If two triangles are congruent, then their areas are the same amount.

Postulate 19: The area of a polygonal region is the sum of the areas of the triangular regions that form the polygonal region (Area Addition Postulate).

Postulate 20: The area of a rectangle is the product of the length of a base and the length of the altitude to that base (A = bh).

Theorem 7-1: The area of a parallelogram is the product of any base and the altitude to that base (A = bh).

Theorem 7-2: The area of a triangle is one-half the product of the length of the base and the altitude to that base (A=1/2 bh).

Corollary 1: The area of an equilateral triangle with side of length s is one-fourth the product of the square of s and the square root of three ($A=1/4s^2 \sqrt{3}$).

Corollary 2: The area of a rhombus is one-half the product of the lengths of its diagonals ($A = 1/2\ d_1d_2$).

Theorem 7-3: The area of a trapezoid is one-half the product of the length of an altitude and the sum of the lengths of the bases (A = 1/2 h

$(b_1 + b_2)$).

Theorem 7-4: The area of a regular polygon is one-half the product of the apothem and the perimeter (A = 1/2ap).

Theorem 7-5: The ratio of the area of two similar triangles is equal to the square of the ratio of the length of any two corresponding sides.

Postulate 21: If two polygons are similar, they can be separated into an equal number of triangles. These triangles will be similar to one nother and in corresponding positions.

Theorem 7-6: The ratio of the areas of two similar polygons is equal to the square of the ratios of the lengths of any pair of corresponding sides.

Theorem 7-7: The ratio of the circumference to the diameter is the same for all circles.

Theorem 7-8: The area of a circle is the product of pi and the square of the radius of the circle ($A = \pi r^2$).

Postulate 22: The volume of any pyramid is one-third the product of the area of the base (B) and the length of the altitude (h) (V = 1/3Bh).

Postulate 23: The volume of a sphere with a radius r is 4/3 π r cubed.($V = 4/3 \pi r^3$)

Postulate 24: The surface area of a sphere with a radius r is $4 \pi r^2$.

($SA = 4 \pi r^2$)